Excercises 9

1)

Let n be a positive integer and $\mathbf{O}(n)$ be the orthogonal group equipped with the standard left-invariant metric

$$g(A, B) = \mathrm{trace}(A^t B).$$

Prove that a C^2-curve $\gamma : (-\epsilon, \epsilon) \to \mathbf{O}(n)$ is a geodesic if and only if

$$\gamma^t \cdot \ddot{\gamma} = \ddot{\gamma}^t \cdot \gamma.$$

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2)

Let the orthogonal group $\mathbf{O}(n)$ be equipped with

the left-invariant metric

$$g(A, B) = \mathrm{trace}(A^t B)$$

and let K be a Lie subgroup of $\mathbf{O}(n)$. Prove that K is totally geodesic in $\mathbf{O}(n)$.

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3)

For the real parameter $\theta \in (0, \pi/2)$ define the 2-

dimensional torus T^2_θ by

$$T^2_\theta = \{(\cos\theta e^{i\alpha}, \sin\theta e^{i\beta}) \in S^3 \mid \alpha, \beta \in \mathbb{R}\}.$$

Determine for which $\theta \in (0, \pi/2)$ the torus T^2_θ is a minimal submanifold of the 3-dimensional sphere

$$S^3 = \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1|^2 + |z_2|^2 = 1\}.$$