. 1) Let S^1 be the unit circle in the complex plane $\mathbb{C}$ given by $S^1 = \{z \in \mathbb{C} \mid |z|^2 = 1\}$. Use the maps $x : \mathbb{C} \setminus \{i\} \to \mathbb{C}$ and $y : \mathbb{C} \setminus \{-i\} \to \mathbb{C}$ with

$$x : z \mapsto \frac{i+z}{1+iz}, \quad y : z \mapsto \frac{1+iz}{i+z}$$

to show that S^1 is a 1-dimensional manifold .

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2)

Show that the composition of smooth maps is smooth.

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3)

Let f and g be real valued functions that are C^∞ on the subsets A and B of M, respectively. Show that $f + g$ and fg are C^∞ on $A \cap B$, where $(f + g)(p) = f(p) + g(p)$ and $(fg)(p) = f(p)g(p)$.

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4)

If $n \geq k$ and g: $R^n \to R^k$ by $g(a_1,\ldots,a_n) = (a_1,\ldots,a_k)$ then g is C^∞ on R^n. If h: $R^k \to R^n$ by $h(a_1,\ldots,a_k) = (a_1,\ldots,a_k, 0,\ldots,0)$ then h is C^∞ on R^k.