Exercises for section 2.1

2.1.1. Reduce each of the following matrices to row echelon form, determine the rank, and identify the basic columns.

(a) $\begin{pmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 8 \\ 2 & 6 & 0 \\ 1 & 2 & 5 \\ 3 & 8 & 6 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 & 1 & 3 & 0 & 4 & 1 \\ 4 & 2 & 4 & 4 & 1 & 5 & 5 \\ 2 & 1 & 3 & 1 & 0 & 4 & 3 \\ 6 & 3 & 4 & 8 & 1 & 9 & 5 \\ 0 & 0 & 3 & -3 & 0 & 0 & 3 \\ 8 & 4 & 2 & 14 & 1 & 13 & 3 \end{pmatrix}$

2.1.2. Determine which of the following matrices are in row echelon form:

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix}$. (b) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} 2 & 2 & 3 & -4 \\ 0 & 0 & 7 & -8 \\ 0 & 0 & 0 & -1 \end{pmatrix}$. (d) $\begin{pmatrix} 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

2.1.3. Suppose that $\mathbf{A}$ is an $m \times n$ matrix. Give a short explanation of why each of the following statements is true.

(a) $rank(\mathbf{A}) \le \min\{m, n\}$.
(b) $rank(\mathbf{A}) < m$ if one row in $\mathbf{A}$ is entirely zero.
(c) $rank(\mathbf{A}) < m$ if one row in $\mathbf{A}$ is a multiple of another row.
(d) $rank(\mathbf{A}) < m$ if one row in $\mathbf{A}$ is a combination of other rows.
(e) $rank(\mathbf{A}) < n$ if one column in $\mathbf{A}$ is entirely zero.

2.1.4. Let $\mathbf{A} = \begin{pmatrix} .1 & .2 & .3 \\ .4 & .5 & .6 \\ .7 & .8 & .901 \end{pmatrix}$.

(a) Use exact arithmetic to determine $rank(\mathbf{A})$.
(b) Now use 3-digit floating-point arithmetic (without partial pivoting or scaling) to determine $rank(\mathbf{A})$. This number might be called the "3-digit numerical rank."
(c) What happens if partial pivoting is incorporated?

2.1.5. How many different "forms" are possible for a 3×4 matrix that is in row echelon form?

2.1.6. Suppose that $[\mathbf{A}|\mathbf{b}]$ is reduced to a matrix $[\mathbf{E}|\mathbf{c}]$.

(a) Is $[\mathbf{E}|\mathbf{c}]$ in row echelon form if $\mathbf{E}$ is?
(b) If $[\mathbf{E}|\mathbf{c}]$ is in row echelon form, must $\mathbf{E}$ be in row echelon form?