decide if the system has a solution. We could conjecture that the left side of a linear system determines the number of solutions while the right side determines if solutions exist, but that guess is not correct. Compare these two systems

$$\begin{aligned} 3x + 2y &= 5 \\ 4x + 2y &= 4 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 5 \\ 3x + 2y &= 4 \end{aligned}$$

with the same right sides but different left sides. The first has a solution but the second does not. Thus the constants on the right side of a system don't decide alone whether a solution exists; rather, it depends on some interaction between the left and right sides.

For some intuition about that interaction, consider this system with one of the coefficients left as the parameter c.

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + y + z &= 1 \\ cx + 3y + 4z &= 0 \end{aligned}$$

If $c = 2$ this system has no solution because the left-hand side has the third row as a sum of the first two, while the right-hand does not. If $c \neq 2$ this system has a unique solution (try it with $c = 1$). For a system to have a solution, if one row of the matrix of coefficients on the left is a linear combination of other rows, then on the right the constant from that row must be the same combination of constants from the same rows.

More intuition about the interaction comes from studying linear combinations. That will be our focus in the second chapter, after we finish the study of Gauss' method itself in the rest of this chapter.

Exercises

✓ **3.15** Solve each system. Express the solution set using vectors. Identify the particular solution and the solution set of the homogeneous system.

(a) $\begin{aligned} 3x + 6y &= 18 \\ x + 2y &= 6 \end{aligned}$ **(b)** $\begin{aligned} x + y &= 1 \\ x - y &= -1 \end{aligned}$ **(c)** $\begin{aligned} x_1 + x_3 &= 4 \\ x_1 - x_2 + 2x_3 &= 5 \\ 4x_1 - x_2 + 5x_3 &= 17 \end{aligned}$

(d) $\begin{aligned} 2a + b - c &= 2 \\ 2a + c &= 3 \\ a - b &= 0 \end{aligned}$ **(e)** $\begin{aligned} x + 2y - z &= 3 \\ 2x + y + w &= 4 \\ x - y + z + w &= 1 \end{aligned}$ **(f)** $\begin{aligned} x + z + w &= 4 \\ 2x + y - w &= 2 \\ 3x + y + z &= 7 \end{aligned}$

3.16 Solve each system, giving the solution set in vector notation. Identify the particular solution and the solution of the homogeneous system.

(a) $\begin{aligned} 2x + y - z &= 1 \\ 4x - y &= 3 \end{aligned}$ **(b)** $\begin{aligned} x - z &= 1 \\ y + 2z - w &= 3 \\ x + 2y + 3z - w &= 7 \end{aligned}$ **(c)** $\begin{aligned} x - y + z &= 0 \\ y + w &= 0 \\ 3x - 2y + 3z + w &= 0 \\ -y - w &= 0 \end{aligned}$

(d) $\begin{aligned} a + 2b + 3c + d - e &= 1 \\ 3a - b + c + d + e &= 3 \end{aligned}$

✓ **3.17** For the system

$$\begin{aligned} 2x - y - w &= 3 \\ y + z + 2w &= 2 \\ x - 2y - z &= -1 \end{aligned}$$