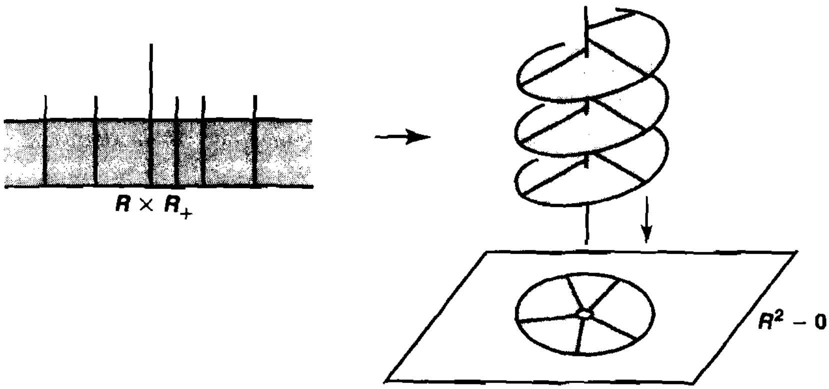

Figure 53.6

Exercises

1. Let Y have the discrete topology. Show that if $p : X \times Y \to X$ is projection on the first coordinate, then p is a covering map.

2. Let $p : E \to B$ be continuous and surjective. Suppose that U is an open set of B that is evenly covered by p. Show that if U is connected, then the partition of $p^{-1}(U)$ into slices is unique.

3. Let $p : E \to B$ be a covering map; let B be connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, then $p^{-1}(b)$ has k elements for every $b \in B$. In such a case, E is called a ***k-fold covering*** of B.

4. Let $q : X \to Y$ and $r : Y \to Z$ be covering maps; let $p = r \circ q$. Show that if $r^{-1}(z)$ is finite for each $z \in Z$, then p is a covering map.

5. Show that the map of Example 3 is a covering map. Generalize to the map $p(z) = z^n$.

6. Let $p : E \to B$ be a covering map.
   (a) If B is Hausdorff, regular, completely regular, or locally compact Hausdorff, then so is E. [*Hint:* If $\{V_\alpha\}$ is a partition of $p^{-1}(U)$ into slices, and C is a closed set of B such that $C \subset U$, then $p^{-1}(C) \cap V_\alpha$ is a closed set of E.]
   (b) If B is compact and $p^{-1}(b)$ is finite for each $b \in B$, then E is compact.

§54 The Fundamental Group of the Circle

The study of covering spaces of a space X is intimately related to the study of the fundamental group of X. In this section, we establish the crucial links between the two concepts, and compute the fundamental group of the circle.