

Figure 60.2

the figure eight in $T\#T$ with the figure eight in Y . Then one retracts Y onto its figure eight by mapping each cross-sectional circle to the point where it intersects the figure eight. Then one maps the figure eight in Y back onto the figure eight in $T\#T$ by the map h^{-1} . ■

Corollary 60.7. *The 2-sphere, torus, projective plane, and double torus are topologically distinct.*

Exercises

1. Compute the fundamental groups of the “solid torus” $S^1 \times B^2$ and the product space $S^1 \times S^2$.
2. Let X be the quotient space obtained from B^2 by identifying each point x of S^1 with its antipode $-x$. Show that X is homeomorphic to the projective plane P^2 .
3. Let $p : E \rightarrow X$ be the map constructed in the proof of Lemma 60.5. Let E' be the subspace of E that is the union of the x -axis and the y -axis. Show that $p|_{E'}$ is not a covering map.
4. The space P^1 and the covering map $p : S^1 \rightarrow P^1$ are familiar ones. What are they?
5. Consider the covering map indicated in Figure 60.3. Here, p wraps A_1 around A twice and wraps B_1 around B twice; p maps A_0 and B_0 homeomorphically onto A and B , respectively. Use this covering space to show that the fundamental group of the figure eight is not abelian.

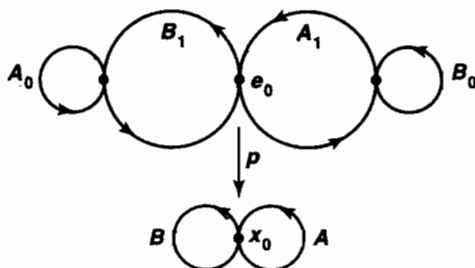


Figure 60.3