In exercises 1 and 2, f_* *means* df and matrix means Jacobi matrix.

1)

Let f: $R^2 \to R^2$ by $f(a, b) = (a^2 - 2b, 4a^3b^2)$ and let g: $R^2 \to R^3$ by $g(u, v) = (u^2v + v^2, u - 2v^3, ve^u)$. Compute a matrix for f_* at $(1, 2)$ and g_* at any (u, v). Find $g_*(4\partial/\partial x - \partial/\partial y)_{(0,1)}$.
Find integral curves for the vector field $X = yi + yj + 2k$ on R^3.

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2)

Let M and N be manifolds with M connected and let f and g be C^∞ maps of M into N. Show $f_* \equiv 0$ iff f is a constant map. If $f(m) = g(m)$ at one m in M and $f_* \equiv g_*$ at all points show $f = g$.

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3)

Show that the map $\pi : TX \longrightarrow X$ is a submersion, hence $\pi^{-1}(p) = T_pX$ is a submanifold of TX.

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4)

Let $X(x) = A \cdot x$ be a linear vector field on $\mathbb{R}^n$, where A is an $n \times n$ matrix. Prove that the flow of X is given by

$$\phi_t(x) = e^{tA} \cdot x, \qquad e^{tA} = \sum_{k=0}^{\infty} \frac{(tA)^k}{k!}.$$

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5)

Compute the Lie bracket of the vector fields $X(x) = (-x_2, x_1, 0)$ and $Y(x) = (x_1x_3, x_2x_3, -x_1^2 - x_2^2)$ on S^2.
(b) Describe the flows of X and Y and explain the result of (a).