1) Let $f: M \to N$ be an injective immersion . Show that if M is compact then $f(M)$ is a submanifold of N.

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2) Let X and Y be manifolds. Prove the following:

a)

Let $y \in Y$, and let $f: X \longrightarrow X \times Y$ be defined by $f(x) = (x, y)$. Then $df_x(v) = (v, 0)$.

b)

Let $f: X \longrightarrow Y$ and $g: X' \longrightarrow Y'$ be smooth maps, and define $f \times g: X \times X' \longrightarrow Y \times Y'$ by $f \times g(x, y) = (f(x), g(y))$. Show that $d(f \times g)_{(x,y)} = df_x \times dg_y$.

c)

Let $f: X \times Y \longrightarrow X$ be the projection. Then

$$df_{(x,y)}: T_{(x,y)}X \times Y \longrightarrow T_x X$$

is also a projection.

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3)

Let X be the subset of $\mathbb{R}^2$ defined by the equation $xy = 0$. Is X a smooth submanifold of $\mathbb{R}^2$?

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4)

Prove that an embedding of a manifold M in a manifold N followed by an embedding of N in a third manifold P is an embedding of M in P.

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5)

Define functions from $\mathbb{R}$ to $\mathbb{R}^2$ as follows: $f(t) = (t, t^3)$, $g(t) = (t^2, t^3)$, $h(t) = (t^3, t^5)$ and $k(t) = (\cos t, \sin t)$.

i. Show that f is a smooth embedding of $\mathbb{R}$ into $\mathbb{R}^2$.

ii. Show that h is a topological embedding of $\mathbb{R}$ into $\mathbb{R}^2$ but not an immersion.

iii. Is g an immersion? An embedding?

iv. Is k an immersion? An embedding?