

1) Compute the $(df)_N$ of antipodal map $f: S^n \rightarrow S^n$ ($f(x)=-x$) at the north pole N.

2)

If M and N are manifolds, then $T_{m,n}(M \times N) = T_m M \times T_n N$ for all $m \in M$ and $n \in N$.

3)

Suppose that $\gamma: \mathbb{R} \rightarrow \mathbb{R}^n$. Show that

$$d\gamma\left(\frac{d}{dt}\right) = \sum_i \gamma^{i'}(t) \frac{\partial}{\partial r_i}.$$

4) Show that

For any tangent vector $v \in T_m M$, there is a curve $\gamma: (a, b) \rightarrow M$ with $\gamma(0) = m$ and $d\gamma_0\left(\frac{d}{dt}\right) = v$.
